

Collective modes as a probe of the equation of state for partially polarized Fermi gases

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We calculate the collective modes of a partially polarized Fermi gas trapped in a spherically symmetric harmonic potential. We show that the breathing mode frequency exhibits non-monotonic dependence on polarization in the entire BCS-BEC crossover region. Moreover, we find that the breathing mode can be used to distinguish between two commonly used unitary gas equations of state.

Recent experimental studies of partially polarized Fermi gases [1, 2] at ultra-cold temperatures near a Feshbach resonance provide a unique window into the behavior of strongly interacting fermions. Relying on long spin relaxation times, experimentalists polarize their two-component Fermi gasses, driving them from superfluid to normal. Consistent with theoretical predictions [3, 4, 5, 6, 7], they see the cloud phase separate into concentric shells, however, the detailed structure of the shells are not completely understood. The Rice experiments [1] have largely been analyzed in terms of a two-shell model where an unpolarized superfluid core is surrounded by a completely polarized normal shell. The MIT experiments [2] strongly suggest a three-shell structure with a partially polarized normal shell between the other regions. There currently exists no reliable model for the equation of state of these partially polarized gases. Here we explore to what extent collective modes can be used to experimentally probe this equation of state.

We use a hydrodynamic approach to calculate the breathing modes of a partially polarized Fermi gas at unitarity. We work with the two most commonly used equations of state [3, 4, 5], finding a significant ($\sim 5\%$) difference between the oscillation frequencies. This sensitivity should be contrasted to the case of unpolarized gases where the differences between the predictions of mean field theories and quantum monte-carlo are much smaller ($\sim 2\%$) [8, 9, 10]. Ongoing experimental efforts which are attempting to observe these smaller beyond mean-field effects [11, 12], could easily distinguish between the two equations of state which we consider, and can help refine our theories of resonant fermions.

We study both the polarization and interaction strength dependence of the breathing mode frequency. We find that the breathing mode frequency is a non-monotonic function of polarization for all interaction strengths.

We consider a zero temperature gas of fermionic atoms of mass m in two hyperfine states $|\sigma = \uparrow, \downarrow\rangle$ confined by a spherically symmetric harmonic potential $U(r) = (m\omega_0^2/2)r^2$, where ω_0 is the trapping frequency. In terms of the numbers of atoms N_σ in each hyperfine state, the polarization is defined as $P = (N_\uparrow - N_\downarrow)/(N_\uparrow + N_\downarrow) \geq 0$. Assuming local equilibrium, the dynamics of this system

will be described by a continuity equation $\partial_t \rho_\sigma + \nabla \cdot \mathbf{j}_\sigma = 0$ and an Euler equation $m\partial_t \mathbf{v}_\sigma + (m/2)\nabla v_\sigma^2 + \nabla U_{\text{eff}} = 0$, where ρ_σ , \mathbf{v}_σ , and $\mathbf{j}_\sigma = \rho_\sigma \mathbf{v}_\sigma$ are the mass density, velocity, and mass current of the atoms in state σ . In the local density approximation $U_{\text{eff}} = U(r) - \mu_\sigma$. Linearizing the equations around equilibrium density, $\rho_\sigma = \rho_\sigma^0 + \delta\rho_\sigma$, and writing the density fluctuations in terms of fluctuations in chemical potential, $\delta\rho_\sigma = \sum_\nu (\partial\rho_\sigma/\partial\mu_\nu)\delta\mu_\nu$, we have

$$\sum_\nu \kappa_{\sigma\nu} \partial_t^2 \delta\mu_\nu = \nabla \cdot \left[\frac{\rho_\sigma}{m} \nabla \delta\mu_\sigma \right]. \quad (1)$$

where we have introduced the compressibility matrix $\kappa_{st} = \partial\rho_s/\partial\mu_t$. Explicitly considering the case of harmonic trapping, the equilibrium local chemical potentials for up and down atoms in the local density approximation (LDA) are given by $\mu_\uparrow(r) = \mu_0 + h - (m\omega_0^2/2)r^2$ and $\mu_\downarrow(r) = \mu_0 - h - (m\omega_0^2/2)r^2$. If we know the equation of state, we can extract the local equilibrium densities from these local chemical potentials. Previous studies have shown that this LDA approach is an excellent approximation unless the number of particles is small and the trap deviates significantly from spherical. Under such circumstances, surface tension between the superfluid and normal regions must be considered [4]. Here we neglect such finite size effects. The parameters μ_0 and h are determined from a constraint on the total number of atoms N and the polarization P .

We formally reduce Eq.(1) to a dimensionless form by defining, $\rho_\sigma = m(m\mu)^{3/2} f_\sigma(\mu/\epsilon, h/\epsilon)$, $\kappa_{\sigma\nu} = m^2(m\mu)^{1/2} g_{\sigma\nu}(\mu/\epsilon, h/\epsilon)$, and $x = \sqrt{m\omega_0^2/2\mu_0}r$, where f_σ and $g_{\sigma\nu}$ are dimensionless functions. The average chemical potential $\mu = (\mu_\uparrow + \mu_\downarrow)/2 = \mu_0(1 - x^2)$ and $\epsilon = \hbar^2/ma_s^2$ with s-wave scattering length a_s and Eq. (1) becomes

$$\sum_\nu g_{\sigma\nu} \left(\frac{\mu}{\epsilon}, \frac{h}{\epsilon} \right) \frac{\partial_t^2 \delta\mu_\nu}{\omega_0} = \frac{\nabla_x}{2} \cdot \left[(1 - x^2)^{\frac{3}{2}} f_\sigma \left(\frac{\mu}{\epsilon}, \frac{h}{\epsilon} \right) \nabla_x \delta\mu_\sigma \right]. \quad (2)$$

We see that dimensionless oscillation frequencies ω/ω_0 only depend on μ_0/ϵ and h/ϵ , or equivalently $\sqrt{2m\mu_0/\hbar^2}a_s$ and P .

We work with two different equations of state: a BCS mean-field result, and a semi-empirical model introduced

by Frederic Chevy [5]. The latter equation of state is sufficiently simple that we can write the solutions of Eq. (1) in terms of hypergeometric functions. The BCS mean field theory is sufficiently complicated that we must solve Eq. (1) numerically. By using spherical symmetry we reduce Eq. (1) to a differential equation for the radial variation of $\delta\mu_\sigma$. After discretizing space and Fourier transforming with respect to time, this radial equation has the form of a matrix eigenvalue problem. Using standard sparse matrix techniques we extract the eigenvalues and eigenvectors. We verify that our results are independent of the discretization procedure.

Neglecting the possibility of a modulated order parameter (FFLO)[13], the mean-field equation of state is found by numerically solving the following equations [3] for the density $\rho = \rho_\uparrow + \rho_\downarrow$, the density difference $\rho_d = \rho_\uparrow - \rho_\downarrow$, and the gap Δ ,

$$\frac{-m}{2\pi\hbar^2 a_s} = \int_0^\infty \frac{d^3\vec{k}}{(2\pi)^3} \left(\frac{1}{E_k} - \frac{1}{\epsilon_k} \right) - \int_{k_-}^{k_+} \frac{d^3\vec{k}}{(2\pi)^3} \frac{1}{E_k} \quad (3)$$

$$\rho = \int_0^\infty \frac{d^3\vec{k}}{(2\pi)^3} \left(1 - \frac{\epsilon_k - \mu}{E_k} \right) + \int_{k_-}^{k_+} \left(\frac{\epsilon_k - \mu}{E_k} \right) \quad (4)$$

$$\rho_d = \frac{1}{(2\pi)^3} \frac{4\pi}{3} (k_+^3 - k_-^3). \quad (5)$$

In these equations, $E_k(\vec{r}) = (E_{k\uparrow} + E_{k\downarrow})/2$, with $E_{k\sigma} = \xi_\sigma h + \sqrt{(\epsilon_k - \mu)^2 + \Delta^2}$, where $\epsilon_k = \hbar^2 k^2/2m$, $\xi_\uparrow = 1, \xi_\downarrow = -1$, $\mu = (\mu_\uparrow + \mu_\downarrow)/2$ and $h = (\mu_\uparrow - \mu_\downarrow)/2$. The momenta of the Fermi surfaces are $k_\pm(\vec{r}) = (\pm\sqrt{h^2 - \Delta^2} + \mu)^{1/2}$. In the normal state, $\Delta = 0$, this approach reproduces the equation of state of a noninteracting Fermi gas. At unitarity, this theory predicts a three-shell structure, similar to that observed at MIT [2], however the intermediate partially polarized normal shell is predicted to be extremely small, and the mean-field theory effectively drops all interactions in this shell. Figure 1 shows the results of this hydrodynamic calculation at unitarity $a_s \rightarrow \infty$, while figure 2 shows the results at several different values of a_s .

Our second approximate equation of state was introduced by Frederic Chevy [5]. Unlike the mean-field theory, this approach assumes that only two phases exist at unitarity: a completely unpolarized superfluid, and a completely polarized normal state. The forms of the equations of state of these two phases are known exactly: in the normal phase, $\rho = B\mu_\uparrow^{3/2}$, while in the superfluid phase, $\rho = A[(\mu_\uparrow + \mu_\downarrow)/2]^{3/2}$. Elementary statistical mechanics gives $B = (1/6\pi^2)(2m/\hbar^2)^{3/2}$, while dimensional analysis requires $A = 2B\varsigma^{-3/2}$. Comparison with experiments and numerical Monte-Carlo calculations give $\varsigma \approx 0.45$ [14]. In this model, the edge of the superfluid corresponds to the edge of the minority cloud (R_\downarrow). At this boundary the ratio of the chemical potential is a universal number, $\mu_\downarrow/\mu_\uparrow = 2(B/A)^{2/5} - 1 = -\xi \approx -0.061$ [4, 5]. The edge of the majority species cloud (R_\uparrow) occurs

when $\mu_\uparrow = 0$, allowing us to write $R_\downarrow^2 = (2/m\omega_0^2)(\mu_0 - h(1 - \xi)/(1 + \xi))$ and $R_\uparrow^2 = (2/m\omega_0^2)(\mu_0 + h)$.

In the superfluid phase all four elements of the compressibility matrix are equal. This structure results from the constraint $\rho_\uparrow = \rho_\downarrow$. In the fully polarized phase, the only nonzero element of the compressibility matrix is $\kappa_{\uparrow\uparrow}$. Thus in each region the compressibility matrix has rank 1, and there is only a single dynamical variable at each point in space. In the superfluid phase ($r < R_\downarrow$), we formalize this observation by taking the dot product (from the left) of the vector $(1, -1)$ and Eq. (1). The time derivative term vanishes and we are left with $\nabla \cdot [(\rho_\uparrow/m)\nabla(\delta\mu_\uparrow - \delta\mu_\downarrow)] = 0$. A sufficient condition for this to be satisfied is $\delta\mu_\uparrow(r) - \delta\mu_\downarrow(r) = \lambda$, where λ is independent of r (for $r < R_\downarrow$). Similarly, in the polarized phase ($r > R_\downarrow$), $\rho_\downarrow = 0$ and we get $\rho_\downarrow \nabla \cdot [(\rho_\downarrow/m)\nabla(\delta\mu_\downarrow)] = 0$. Again, a sufficient condition is $\delta\mu_\downarrow(r > R_\downarrow) = s$, where s is a constant. Motion of the boundary is captured by the time dynamics of s and λ .

To match the solutions in the two regions we note that at the edge of the minority cloud, the density is discontinuous. This implies that the compressibility matrix has a delta-function singularity. Including this singularity in Eq. (1), one sees that the quantity in the divergence, $f_\sigma = \nabla\delta\mu_\sigma/\rho_\sigma$, is discontinuous. By integrating Eq. (1) in the neighborhood of R_\downarrow , one finds that this discontinuity is due solely to the discontinuity in ρ_σ , and that both $\delta\mu_\uparrow$ and its first derivative are continuous.

Equating the two expressions for $\delta\mu_\downarrow$ at the boundary, we have $\delta\mu_\uparrow(R_\downarrow) - s = \lambda$. Introducing $w(r) = \delta\mu_\uparrow(r) - \delta\mu_\uparrow(R_\downarrow)$, we then have,

$$\partial_t^2(w + s + \lambda/2) = \frac{1}{2\kappa_{\uparrow\uparrow}} \nabla \cdot \left[\frac{\rho_\uparrow}{m} \nabla w \right], \quad r < R_\downarrow, \quad (6)$$

$$\partial_t^2(w + s + \lambda) = \frac{1}{\kappa_{\uparrow\uparrow}} \nabla \cdot \left[\frac{\rho_\uparrow}{m} \nabla w \right], \quad r > R_\downarrow. \quad (7)$$

Subtracting Eq. (6) from (7) and setting $r = R_\downarrow$ we get an ordinary differential equation for λ , which is readily solved in terms of $w(r = R_\downarrow)$, allowing us to eliminate λ from Eq. (6) and (7). Finding the compressibilities from the equation of states, we derive a closed set of equations for $r > R_\downarrow$ and $r < R_\downarrow$ respectively.

$$\frac{\partial_t^2 \delta\mu_\uparrow(r)}{\omega_0^2} = \frac{(R_\uparrow^2 - r^2)}{3} \nabla^2 \delta\mu_\uparrow(r) - r \partial_r \delta\mu_\uparrow(r) \quad (8)$$

$$\frac{\partial_t^2 \delta\mu_\uparrow(r)}{\omega_0^2} = \frac{(\delta^2 - r^2)}{3} \nabla^2 \delta\mu_\uparrow(r) - r \partial_r \delta\mu_\uparrow(r) + X.$$

where $\delta^2 = R_\uparrow^2/(1 + h/\mu_0)$ and $3X/2 = R^2 \nabla^2 \delta\mu_{\uparrow+}(R_\downarrow) - \Gamma^2 \nabla^2 \delta\mu_{\uparrow-}(R_\downarrow)$. Here $R^2 = R_\uparrow^2 - R_\downarrow^2$, $\Gamma^2 = \delta^2 - R_\downarrow^2$, and $\delta\mu_{\uparrow+/-}(R_\downarrow)$ is the fluctuation of the majority species chemical potential on the normal/superfluid side of the boundary. Assuming $\delta\mu_\uparrow \propto \exp[i\omega t]$, and using a suitable change of variables, each of the equations in Eq. (8)

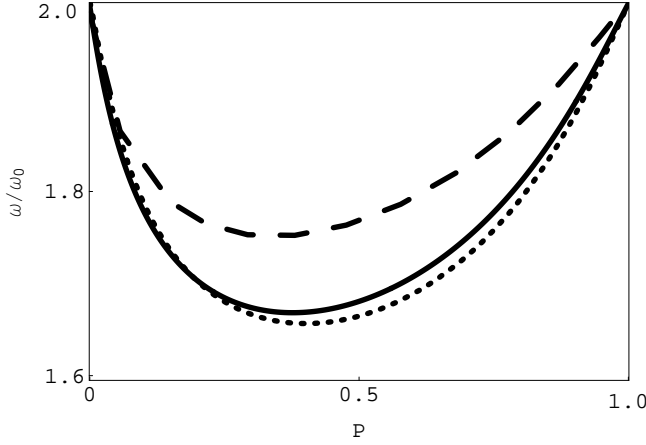


FIG. 1: Breathing mode frequencies of a spherical two component Fermi gas as a function of polarization P at unitarity. Solid line: From matching the analytic solutions at the boundary between superfluid and normal region in the two shell approximation. Dotted line: From numerical solutions of Eq. (1) using the two-shell equation of state. Dashed line: Numerical solutions of Eq. (1) in the BCS approximation (which predicts a three shell structure).

can be converted into hypergeometric equations and the solutions are given in terms of Hypergeometric functions $F(a, b; c, u)$ as [15],

$$\delta\mu_{\uparrow} = C_1 F(\alpha, \beta; \eta, 1 - r^2/R_{\uparrow}^2) r^l Y_{l,m}(\theta, \phi) \quad (9)$$

$$\delta\mu_{\uparrow} = -\bar{X} + C_2 F(\alpha, \beta; \gamma, r^2/\delta^2) r^l Y_{l,m}(\theta, \phi) \quad (10)$$

for $r > R_{\downarrow}$ and $r < R_{\downarrow}$ respectively. The parameters $\alpha = (1/2)[2 + l + \sqrt{4 + 3\epsilon + l^2 + l}]$, $\beta = (1/2)[2 + l - \sqrt{4 + 3\epsilon + l^2 + l}]$, $\gamma = (2l + 3)/2$, and $\eta = \alpha + \beta + 1 - \gamma$ are functions of magnetic quantum number l and the dimensionless mode frequencies $\epsilon = \omega^2/\omega_0^2$. Here $\bar{X} = (2/3)[R_{\downarrow}^2 C_1 G_1(R_{\downarrow}) - \Gamma^2 C_2 G_2(R_{\downarrow})] R_{\downarrow}^l Y_{l,m}(\theta, \phi)$. Notice, we introduced two arbitrary constants C_1 and C_2 . The functions G_1 and G_2 are defined as, $G_1(R_{\downarrow}) = (a/b)F(\alpha + 2, \beta + 2; \eta + 2, 1 - R_{\downarrow}^2/R_{\uparrow}^2) - (c/d)F(\alpha + 1, \beta + 1; \eta + 1, 1 - R_{\downarrow}^2/R_{\uparrow}^2)$ and $G_2(R_{\downarrow}) = (a/e)F(\alpha + 2, \beta + 2; \gamma + 2, R_{\downarrow}^2/\delta^2) + (c/(\gamma\delta^2))F(\alpha + 1, \beta + 1; \gamma + 1, R_{\downarrow}^2/\delta^2)$ [16].

We match the two solutions and their derivatives at the boundary to get the frequencies ω for the breathing modes ($l = 0$) at unitarity. The lowest energy breathing mode frequency is shown in Fig. 1 as a function of polarization. We compare these results with those from the BCS equation of state. We also numerically solve Eq. (1) using the two-shell equation of state. The slight deviations of numerical and analytic results obtained using the two shell equation of state are numerical artifacts due to the smearing out of singularities in the compressibilities at the boundary.

At both $P = 0$ and $P = 1$ the whole cloud is in a single phase with an equation of state of the form $n \propto \mu^{3/2}$. Consequently, in both of these limits the cloud

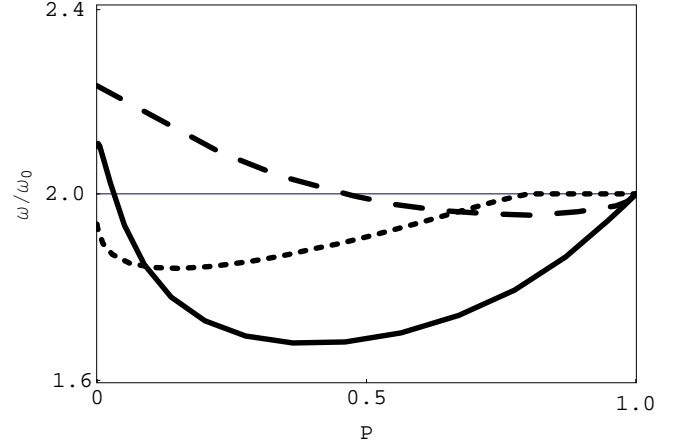


FIG. 2: Numerically calculated breathing mode frequencies in the BCS approximation. Dotted line: BCS regime where theory predicts a two/three shell structure; superfluid, partially polarized normal and fully polarized normal shells. Solid line: Crossover-BEC regime where theory predicts a two shell structure; superfluid and fully polarized normal shells. Dashed line: Deep BEC regime where theory predicts a two/three shell structure; superfluid, polarized superfluid, and fully polarized normal shells. These three graphs use the dimensionless parameter $\sqrt{2m\mu_0/\hbar^2}a_s = -2.0, +2.0, +0.2$ respectively.

breathes with frequency $\omega = 2\omega_0$ [9]. For intermediate polarizations, the mismatch of the speed of sound causes a drop in the oscillation frequency.

Away from unitarity we must rely on the mean-field calculations, as we know of no non-unitary analogy of Chevy's equation of state. In the BCS regime ($a_s < 0$), mean-field theory predicts a three-shell structure at low polarizations: an unpolarized superfluid core surrounded by partially polarized normal, and fully polarized normal shells. At higher polarizations, the superfluid core is absent. Since in the absence of a superfluid region the mean-field equation of state reduces to that of non-interacting particles, one finds that at these large polarizations the breathing mode frequency again becomes $\omega = 2\omega_0$ (see FIG. 2). In the deep BEC limit ($a_s > 0, k_f a_s \ll 1$), the mean-field theory again predicts a three-shell structure: an unpolarized superfluid core surrounded by a partially polarized superfluid shell and a fully polarized normal shell. At high polarizations the inner core will be absent. In this regime we do not see any dramatic signature of the disappearance of the central superfluid core.

As seen in FIG. 2, the qualitative behavior of the polarization dependence on breathing modes has the same non-monotonic behavior in the entire BCS-BEC crossover region.

In ref. [8], we used a sum rule approach to find collective mode frequencies for unpolarized Fermi gases. Although the sum rule technique only provides upper

bounds on these frequencies, we found that the bound was very tight, and that method produced excellent agreement with both experiments [11, 12] and hydrodynamic theories [9, 10]. Repeating those calculations for the partially polarized gas, we find that the sum rules provide a much weaker upper bound in the present case. For example, using the two-shell equation of state at unitarity, the sum rule calculation finds no polarization dependence. It simply bounds $\omega \leq 2\omega_0$ for all P .

Although here we only consider spherically symmetric traps, we believe that the qualitative behavior of the polarization dependence of axial and radial breathing modes in an anisotropic traps will be similar to what we found. In particular the frequencies of these modes should be sensitive to the equation of state. We speculate that in highly asymmetric traps, such as those used at Rice, surface tension effects may begin to play a role in the collective mode frequencies, though we have not calculated these effects.

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